

# Weekly Homework 2

Math 485

September 18, 2013

## 1 Textbook (Stampfli and Goodman)

Section 2.2, Page 29: 1,2,3,4.

Section 2.4 Page 35: 1,2,5,6.

## 2 Additional problems

1. (Extra credit - 5 pts) In this problem we will verify that the conditional expectation  $E(X|Y)$  is the best guess of  $X$  given  $Y$  in the following sense

$$E[(X - E(X|Y))^2] \leq E[(X - g(Y))^2], \text{ for all } g(Y). \quad (1)$$

a. Show that

$$E[E(X|Y)X] = E[E(X|Y)^2].$$

Hint:

$$E[E(X|Y)X] = E\left[E\left(E(X|Y)X \mid Y\right)\right].$$

Proceed using properties of conditional expectation.

b. Use the result of part a to prove (1).

2. Suppose Bob is long one European put on XYZ stock at strike  $K_1$  and short another put on XYZ at a different strike price  $K_2$ , where  $K_2 < K_1$ , but with the same expiration date. Graph the payoff to Bob at the time of expiration. Bob's position is called a *bear spread*. Explain this terminology, understanding that a *bear market* is a declining market.

3. (a) Suppose Apple is currently trading at \$500.00. Alice believes Apple will go up in a major way and wants to bet \$60,000 on this speculation. Right now, suppose a January call at strike \$600 costs \$6.00. Therefore Alice can buy 10,000 calls with her money; since Apple options are exchange-traded in standardized contracts for the purchase or sale of 100 shares, this means she can buy 100 contracts. Suppose she buys and holds the option until expiry. Call this investment strategy I. Strategy II is to simply invest the entire \$60,000 today in Apple stock and sell in January at the same date the options expire. (For simplicity assume the interest rate is 0.) If the stock price on the date of expiry is \$620 per share, calculate her profit from each different strategy.

**Comment:** You should find in (a) that the profit for strategy I is significantly greater; this is an example of how an option may provide leverage for speculation. Of course, strategy I also magnifies possible losses. If the option expires worthless, then Alice will lose her entire \$60,000. She would lose everything in strategy II only if the stock price went to 0, which is very unlikely.

(b) For each strategy in (a), determine the profit or loss as a function of the price  $S_T$  of Apple stock at the expiration date. Determine that price  $S_T^*$  at which both strategies produce the same profit. Alice would have to strongly believe that  $S_T$  will be larger than  $S_T^*$  to prefer strategy I.

4. A hedger would buy a put on XYZ stock to protect against an increase in XYZ stock price. A speculator would buy a put on XYZ stock to try to profit from a decrease in XYZ stock price. Explain.

5. Suppose you enter a 3 month forward contract on a non-dividend paying stock. The price of the stock today is \$ 25 and the risk-free borrowing and lending rate is 6% per annum. What is the forward price?

A month later the price of the stock is \$ 30 and the interest rate is the same. What is the above contract worth at this time?

6. In this question, we will find the forward price for a forward contract on a stock that pays dividend. Assume the interest rate is for the money market is  $r$ .

a. Suppose a stock pays a dividend amount  $d$  at time  $t$ , where  $0 < t < T$ . Namely, at time  $t$ , the holder of 1 share of stock receives  $d$  dollars. Shows that the forward price is

$$F(0, T) = [S_0 - e^{-rt}d]e^{rT}.$$

b. Suppose a stock pays dividend continuously at a rate  $r_d > 0$ , called the dividend yield, which is then reinvested in the stock. Namely, an investment in one share held at time 0 will increase to become  $e^{r_d T}$  at time  $T$ . Show that the forward price is

$$F(0, T) = S_0 e^{(r-r_d)T}.$$